# Investigating the relation between merger times, masses and spins of binary black hole and binary neutron star coalescences

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28/07/2020

Pioneer Academics SP20, under the supervision of Professor Eric Myers

## Abstract

This paper looked for correlations between masses and spins of binary black hole events and their merger times. By using data released by LIGO and tutorial 2.2 from the Open Data Workshop, I got merger times, mass and spin data, which I plotted using Matplotlib in Python. I found moderate correlations between masses, effective inspiral spin and merger times, and very weak correlations between mass ratios, percentage change in mass, final spin and merger times.

## I. Introduction

Since the first direct detection of gravitational waves, the signals received have given new data about black holes and neutron stars and, ushered in a new era of physics and astronomy. Using tutorials available on the Gravitational Wave Open Science Center (GWOSC) website, my objective is to calculate the merger times and see if there is a correlation between merger times and masses and spins.

I chose to study merger times over inspiral times because defining the beginning of an inspiral and taking into account the distances of events from earth impacting inspiral times proved to be difficult for the duration of this program. Moreover, while there is research about merger times, there is no definite study of correlation between masses and merger times and spins and merger times, and I wanted to see if I could find one or show that one does not exist.

## III. Background

### A. Gravitational Waves

Gravitational waves were first proposed in 1916. Since then, many efforts have been made to prove their existence. They are ripples that travel at the speed of light in the fabric of spacetime, proposed by Albert Einstein in his theory of general relativity [1]. Spacetime includes the three physical dimensions and the fourth dimension, time; gravitation in spacetime is caused by the curvature caused by objects in it. When a large-scale, highly energetic event with a changing quadrupole (or higher pole) moment occurs, it causes ripples in spacetime, also called gravitational waves [2].

Gravitational waves alternatingly compress an object in one dimension and stretch it the other. Depending on the polarization of the gravitational wave, the object will stretch and compress in different ways [3]. Gravitational waves have only two independent polarizations according to general relativity: '+' and '×', though other theories of gravitation predict that there can be six different polarizations [4]. These waves have been divided into four categories [5] based on how they are produced, namely continuous, compact binary inspiral, burst and stochastic.

Continuous gravitational waves are produced by a system with a definite frequency. They may be caused by, for example, a spinning neutron star with a deformation or irregularity. Gravitational waves of this type are expected to give weaker signals [6]. As the spin time of an object that produces these waves stays constant, so do the gravitational waves created [7].

Another source of gravitational waves is a compact binary inspiral. This consists of two objects whose orbits are reaching their end stage, that is, their orbital radii are decreasing as they come closer and closer together, eventually colliding. These systems have three possibilities: binary neutron stars (BNS), binary black holes (BBH), or a black hole and a neutron star (BH-NS). At the end, they form a single object [8].

A third type of gravitational wave forms the stochastic gravitational wave background. Many events that produce gravitational waves occur too far from the earth for the detectors to get a very strong, directed signal. Instead, the far-off waves all seem to merge into one single stochastic (random) gravitational wave background [9]. These are most likely remnants of gravitational events from the early universe. If detected, this background, which is similar to the cosmic microwave background, can provide information about the universe in its initial stages [10]. In fact, if these waves actually are from the Big Bang, they may provide more information than even the cosmic microwave background, as the CMB formed about 300,000 years after the Big Bang but these waves would have been produced less than a second after it [11].

The final type of gravitational wave is one that we cannot predict – burst. Bursts refer to short duration gravitational waves that may form from events such as supernovae or gamma ray bursts [12].

## B. Detection

While theory predicted gravitational waves in 1916, the first indirect proof for them came in in 1974, when Joseph Hooten Taylor and Alan Russell Hulse found the first pulsar binary in the sky. While observing it, they found that their orbiting motion (an acceleration and changing quadrupole moment) released gravitational energy [13]. Comparing their results to Einstein's predictions, they found that general relativity passed this test with a difference of less than 0.4%. This suggested that gravitational waves exist, are quadrupole in nature and travel at the speed of light [14]. This work won them the Nobel Prize in Physics in 1993.

Direct detection of gravitational waves was problematic because the waves alter objects by very small distances. In 1960, Joseph Weber came up with an instrument to try and detect gravitational waves, and by 1966, he and his team finished building it. In 1969, he announced that he had detected gravitational waves. However, his results were not accepted. Still, he is considered to be the pioneer of gravitational-wave detection, and his instruments, the Weber bars, are named after him [13].

In the late 1960s, interferometric gravitational wave detectors were built with mounted mirrors, followed by swinging mirror interferometers in the 1970s. In 1987 a proposal was put forth to build two, four-kilometer-long interferometers with a two-step approach – initial interferometers followed by advanced interferometers. By 1992, two sites, Hanford and Livingston, had been chosen. By 2002, construction was finished, and the Laser Interferometer Gravitational Wave Observatory started its first science run, that is, its first search for gravitational wave signals [15]. In 2007, a few changes were made at the two LIGO facilities, which included increasing laser power and adding in-vacuum hardware, amongst others [16]. There were no detections after this was complete. Then, in 2011, advanced LIGO was set up. Work continued until 2015. Finally, on September 14, 2015, LIGO detected its first gravitational wave. Results were confirmed and announced at the beginning of 2016 [15]. In 2017, Barry C. Barish, Rainer Weiss and Kip S. Thorne won the Nobel Prize in Physics for LIGO and the detection of gravitational waves [17].

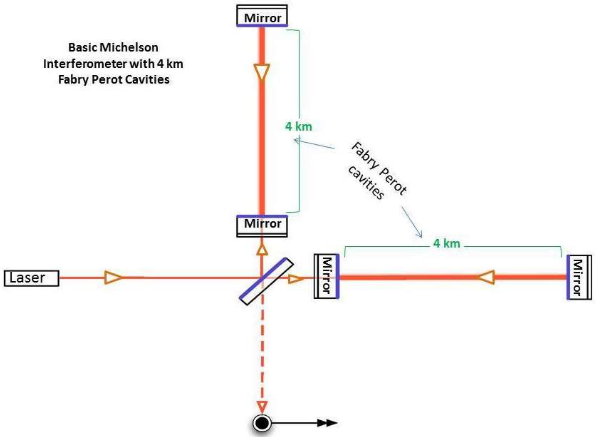

Fig.1 A diagram showing a basic Michelson interferometer with Fabry-Perot cavities [18]

The LIGO detectors are essentially Michelson interferometers. They divide a laser beam into two halves, perpendicularly, and shoot them at mirrors which are 4 km away. A Fabry-Perot cavity has been created within those two arms where the beam is reflected, that is, another mirror causes the laser beam to reflect about 300 times, vastly increasing its sensitivity (the distance the beam travels goes from 4 km to 1200 km). Then the two split beams recombine and are sent to a detector [18]. If the distance the two beams have traveled is exactly the same, then they will add, so the sensors will report light.

However, if a gravitational wave passes through, it will stretch one arm and compress the other (as the two arms are perpendicular to each other). This will cause the recombined beams to cancel out, giving different patterns of light. This light is then converted to give the strain (how much stretching was caused) value of the wave that passed through [19].

Still, LIGO faces challenges from sources of noise that may cause erroneous detections. It can be affected by things such as airplanes flying overhead, seismic vibrations and even people walking by it. To isolate the system from these potential sources of noise, advanced LIGO uses seismometers, fused silica fibers to suspend their mirrors (making the mirrors act as a pendulum), and places the mirrors in boxes that travel in the opposite direction if the ground ever moves, making the mirrors relatively still [20].

In addition, other observatories also exist: Virgo, GEO600 and KAGRA. Virgo is situated in Italy, GEO600 is located near Hannover in Germany and KAGRA in Japan [21] [22].

More detectors are planned for the future. There is a planned detector in India (LIGO-India). LISA (Laser Interferometer Space Antenna) is a planned mission to have a gravitational wave detector in space, allowing data from all over the universe to be collected. This will consist of three spacecraft, whose distance in space will cause LISA’s arms to be around 2.5 million km long [23].

## C. Compact Binary Inspirals

Objects in a compact binary coalesce towards each other and form either a single black hole or neutron star. While they are far apart, they orbit each other slowly and generate weaker gravitational waves, which travel through spacetime and cause a loss of energy in the system. This loss of energy causes a radiation reaction, which is a decrease in the orbital radius and an increase in the orbital frequency. As a consequence, the black holes or neutron stars orbit faster and release gravitational waves with more energy, causing an accelerated decrease in the size of the system. This is called the inspiral phase. In the merger phase, the two objects get close enough to merge into a single one. This phase typically lasts for less than a second. This is followed by the ringdown phase, which gives a decreased strain signal [8].

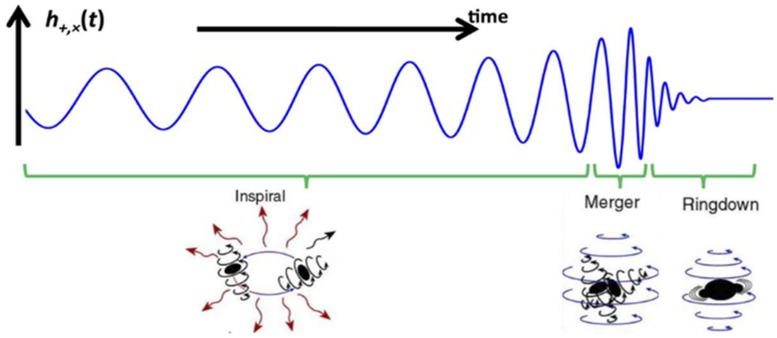

Fig. 2 A graph showing the three phases of a binary coalescence [8]

D. Spin

Objects such as black holes and neutron stars spin in space.

For a black hole, the effective spin is a dimensionless quantity ranging from -1 to 1 that measures the spin of the spacetime that is dragged into it with the addition of the angular momentum of everything that went into the black hole [24]. A neutron star spins due to a conservation of angular momentum – as the star shrinks, its spinning speeds up [25].

For coalescing binaries of black holes and neutron stars, two spin values are calculated: final spin and effective inspiral spin. Final spin is the spin of the single body that forms after merger. Effective inspiral spin is a measure of how well-aligned the bodies' spins are to their orbital axis. It is given by this equation:

$$X_{eff} = \frac{m_1 a_1 \cos\theta_{LS_1} + m_2 a_2 \cos\theta_{LS_2}}{M} \quad \text{(Equation 1)}$$

While it is known that effective inspiral spin has an impact on inspiral time, some suspect that it may also affect merger times. [26]

## IV. Methodology

Though I started my project using the Binary Black Hole Event Tutorial [27] on the GWOSC website [28], I switched to tutorial 2.2 (in a Google Colab notebook) of the Open Data Workshop (26 to 27 May, 2020) [29], because tutorial 2.2 employed the use of the PyCBC python module that directly imported data from LIGO's event catalogue, eliminating the need to create JSON files and templates individually for all gravitational wave events.

PyCBC is software used to inspect gravitational wave sources, containing algorithms for detection and parameter estimation of coalescing binaries. [30]

Tutorial 2.2 begins by getting data of an event from a specified detector. Using matched filtering (a comparison of detector data to templates made using theory), it then calculates the power spectral density (a measure of the signal's power at a given frequency). Utilizing the waveform approximant `SEOBNRv4_opt` [31], it calculates the signal-to-noise time series. After this, it creates a graph of strain against time, giving the user the ability to define the start and end time of the graph by setting them as seconds before and after the merger.

By setting the start time at 0.0 seconds before merger, that is, at the beginning of the merger, I was able to start my graphs at exactly the time of the start of the merger. I used PyCBC's definition of the start of merger. After setting the end time at 0.1 seconds after merger, I used the `numpy().argmax()` function to find the index value of the maximum strain (which signifies the end of the merger time). Using that index value, I identified both the value of maximum strain and the time at which it occurred. Furthermore, by printing the merger time and the time at which maximum strain occurred, I got the beginning and end times of the merger for each event. To get the duration of merger, I subtracted the beginning time from the end time.

There were differences in the merger times from H1 (Hanford detector) and L1 (Livingston detector). To account for those, I took the average of the merger times from both detectors' data.

Additionally, I used the Event List on the GWOSC website [32] and the data releases for each event to get access to all the data about mass, specifically the data of mass 1 (the bigger mass in each merger), mass 2 (the smaller mass in each merger), remnant mass (the mass left after the coalescence is complete and energy is lost) and chirp mass (the effective mass of a binary system in terms of gravitational radiation given out by it [33]) and spin, specifically effective inspiral spin and final spin. I initially used tutorial 2.4 from the same workshop to get the masses by doing a parameter estimation; however, as there were differences in the values I received and official ones by LIGO, I decided to use the LIGO values for more accuracy.

Next, using Python (`matplotlib.pyplot` and `numpy` modules), I plotted graphs of each set of mass data and spin data against merger time. I then used `scipy.stats.pearsonr` to calculate the Pearson correlation coefficient (a measure of how much correlation exists between two variables) for each set of data.

## V. Results

### A. Mass

I plotted graphs of merger time against mass 1, mass 2, remnant mass, chirp mass, mass ratio and the percentage change in total mass.

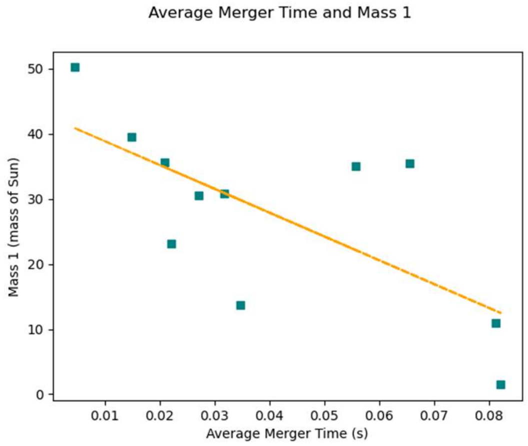

Fig. 3 Mass 1 against average merger times

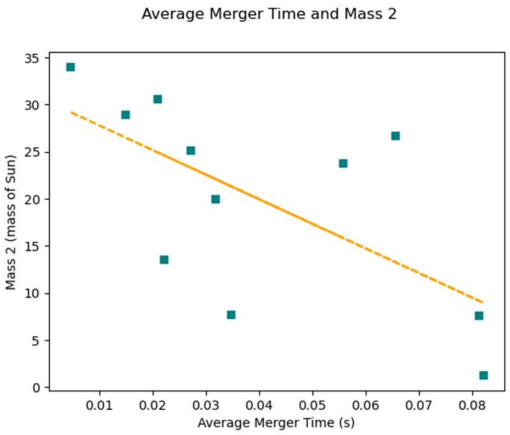

Fig. 4 Mass 2 against average merger times

## A.1 Mass 1 and Mass 2

The correlations between average merger time and mass 1 and mass 2 have a Pearson coefficient of -0.691 and -0.648 respectively, indicating a moderate negative correlation between these parameters and merger times. Fig. 3 and Fig. 4 show the two graphs.

Seven out of eleven merger times of coalescing binaries are between 0.00 and 0.04 seconds. If looked at in isolation, there is a strong negative correlation between mass 1 and merger times and mass 2 and merger times in this interval.

There is no event with a merger time between 0.04 and 0.05 seconds, and only four events with merger times of more than 0.05 seconds. There is also no merger time that goes up to or beyond 0.09 seconds.

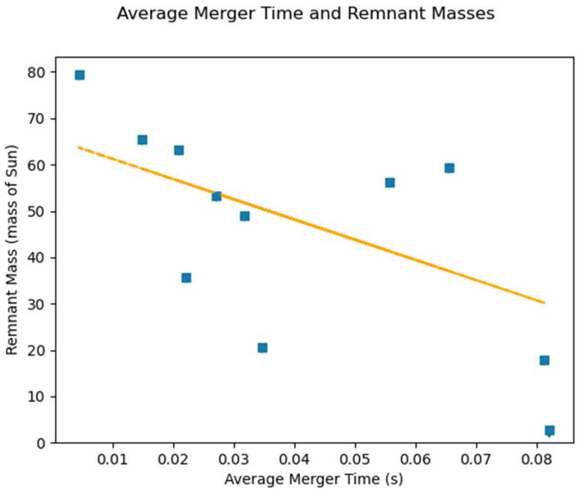

Fig. 5 Remnant masses against average merger times

### A.2 Remnant Mass

Fig. 5 shows the graph of remnant mass against time. The Pearson coefficient for remnant mass and merger time is -0.532, indicating a moderate correlation between both, and one that is smaller than that of mass 1 and mass 2 with merger time. There is also a strong correlation for merger times in the interval of 0.00 and 0.04 seconds.

In this graph, for GW170817, the remnant mass of the neutron star binary was never detected [34], so the remnant has been plotted at 2.8 solar masses with an arrow pointing downwards to show the uncertainty.

The points on this graph are similar to the graphs of mass 1 and mass 2 in Fig. 3 and Fig. 4 respectively, as remnant mass depends on the initial masses to a degree.

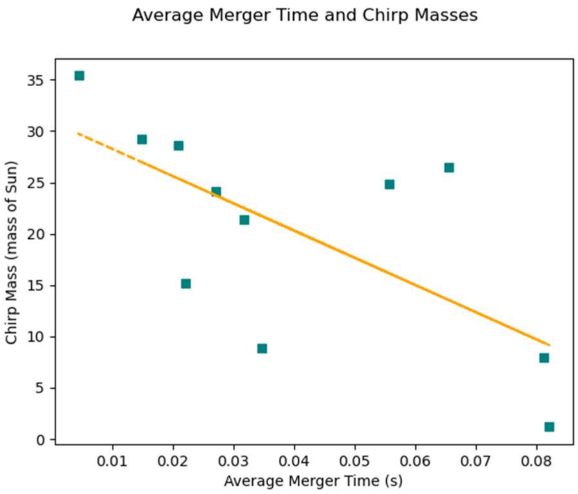

Fig. 6 Chirp masses against average merger times

## A.3 Chirp Mass

Chirp mass is given by the equation [35]:

$$M_{ch} = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}} \qquad \text{(Equation 2)}$$

As evidenced by the equation above, chirp mass also depends on the two coalescing masses in a binary. Fig. 6 shows the graph of chirp mass against average merger time. The Pearson coefficient value for this graph is -0.671, showing a moderate correlation between chirp mass and average merger time.

The placement of points on the graph in Fig. 6 is almost identical to the one in Fig. 5, save for the inclusion of GW170817. This is because both remnant mass and chirp mass depend on the same initial masses – mass 1 and 2 – ensuring that the event with the largest remnant mass also has the largest chirp mass.

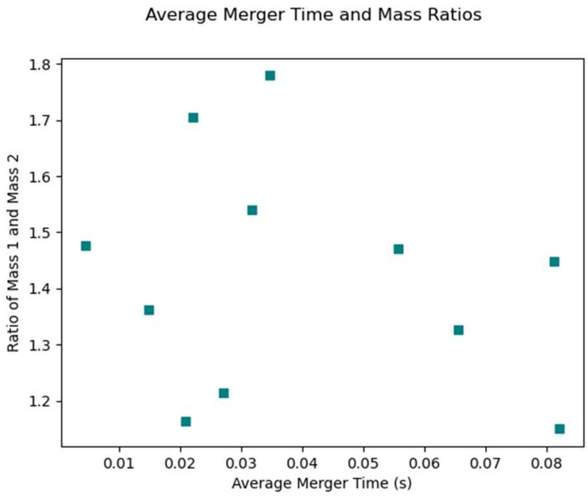

Fig. 7 Mass ratio against average merger times

### A.4 Mass Ratio

The graph in Fig. 7 shows the ratios of mass 1 and mass 2 against average merger times, where I have taken the ratio to be:

$$ratio = \frac{m_1}{m_2} \qquad \text{(Equation 3)}$$

There is very little correlation between the ratio of the masses and merger times, with a Pearson coefficient value of -0.232, meaning merger times are not impacted by the difference between masses.

Another thing to note is that for all the events in this investigation, the ratios of mass 1 and mass 2 are between 1.1 and 1.8, further confirming the fact that most events' inspiralling masses are very close in value. However, this also means that the data set is very limited and further investigation with new data beyond this range might yield new results.

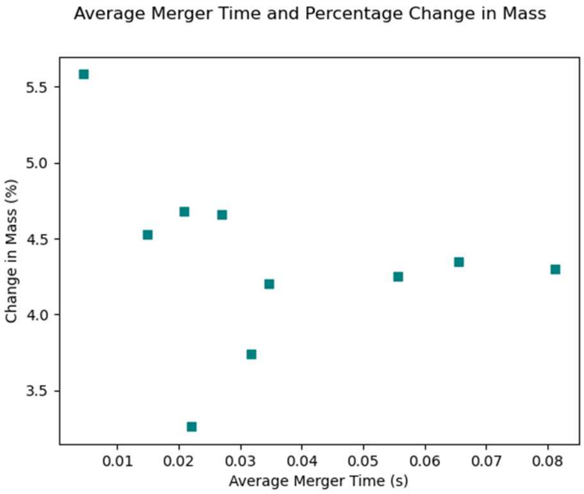

Fig. 8 Percentage change in mass against average merger times

## A.5 Change in Mass

I found the percentage change in total mass using the equation:

$$change = \frac{(m_1+m_2)- m_{remnant}}{m_1+m_2} \times 100 \qquad \text{(Equation 4)}$$

The graph in Fig. 8 shows percentage change in mass against average merger time. The graph has a Pearson coefficient of -0.277, showing very weak correlation, meaning merger time is likely to be almost completely independent of the change in total mass. GW170817 is not included in this graph as its remnant mass has not been detected.

The percentage change in total mass ranged between 3.2 and 5.6, with the majority of points lying between 4.0% and 5.0%. This means that most events have similar loss of mass.

## B. Spin

I made graphs of final spin and effective inspiral spin against merger time. Unlike mass, I have not taken averages of the final spin and effective inspiral spin uncertainty values because the uncertainty values are much larger than the uncertainty values of masses. Here, the error bars show the uncertainty values reported by LIGO.

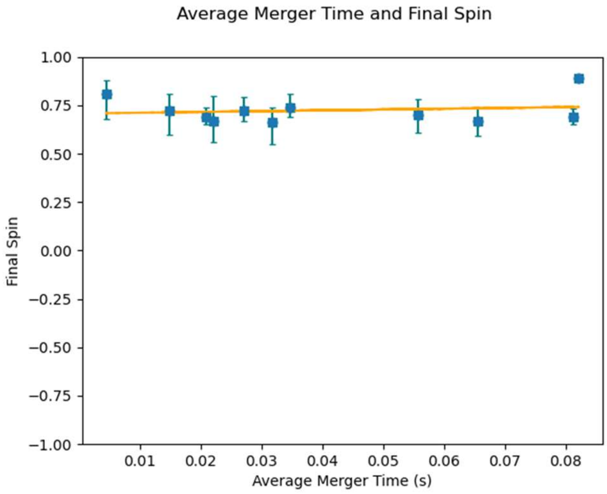

Fig. 9 Final spin against average merger times

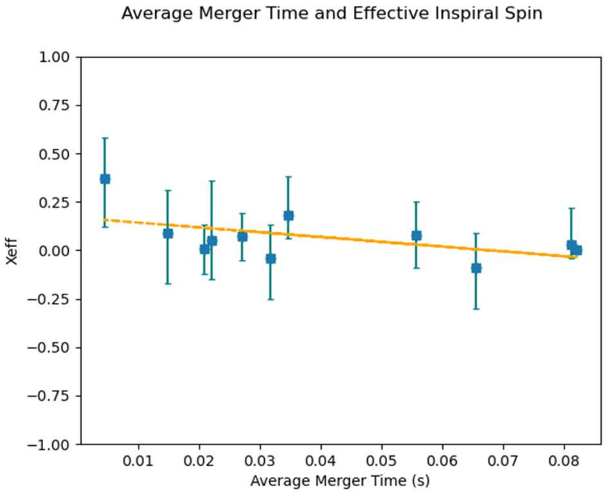

Fig. 10 Effective inspiral spin against average merger times

## B.1 Final Spin

The graph in Fig. 9 shows the final spins against average merger times for each event. The Pearson coefficient for this is 0.161, signifying a very weak correlation between final spins and average merger times and showing that merger times are likely to be independent of final spin.

For GW170817, the final spin data by LIGO was given as ≤ 0.89, so for this graph, I used 0.89 as the value for final spin. Additionally, the graph shows that the final spins detected for each event are positive, and no event with a negative final spin has been detected yet.

## B.2 Effective Inspiral Spin

As stated before, the effective inspiral spin is a measure of the alignment of bodies' spin to their orbital axis while they are orbiting each other. Effective inspiral spin is given by Equation 1:

$$X_{eff} = \frac{m_1 a_1 \cos\theta_{LS_1} + m_2 a_2 \cos\theta_{LS_2}}{M}$$

In this equation, $m_1$, $m_2$ and $a_1$, $a_2$ are the masses and spins of the two objects respectively. $\Theta_{LS1}$ and $\Theta_{LS2}$ are the tilt angles between the spin and orbital angular momentum of the two inspiralling objects.

The uncertainties for effective inspiral spin data were much larger than any other mass or final spin data, ranging from 0.04 to 0.30 above and below the values.

The graph in Fig. 10 shows a negative correlation between effective inspiral spin and merger time, with a Pearson coefficient of -0.542, signifying a moderate correlation. This shows that while effective inspiral spin may affect inspiral time [26], it is not likely to impact merger time to a great degree.

## VI. Conclusions

Through this investigation of mass, spin and merger times, a number of conclusions can be drawn.

Firstly, merger time seems to have a small negative correlation with mass 1, mass 2, remnant mass and chirp mass, with a stronger correlation between 0.00 and 0.04 seconds of merger time. Of the eleven O1 and O2 events used in this paper, seven have merger times in this interval. If further analysis is done, using more data, for example data from observation run 3 of the LIGO detectors, a stronger relation may be found.

Additionally, there seems to be no correlation between the ratios of the two masses that are merging and the merger time.

Secondly, for spin, there is a slight negative correlation of merger time with final spin, suggesting that final spin does not impact merger time.

Furthermore, the correlation between effective inspiral spin and merger time is weak and negative, indicating that it has a very small, if any, impact on merger time.

## VII. Further Research

Data from the third observation run (O3) of the LIGO detectors was released late during the process of this program, so it could not be included in this paper. However, future investigations could include O3 data to increase their data set and get stronger analysis and correlations. Initial O3 events look promising for an investigation of this kind, especially GW190814, which had masses that differed by a factor of around 9, something that has not been seen by LIGO before [36].

## VIII. Summary

In this paper, I studied the correlations of masses and spins with merger times of coalescing binaries. Using Tutorial 2.2 of the Open Data Workshop in May 2020, I calculated the merger times for each of the eleven events from O1 and O2 of LIGO. I used the event catalogue on the GWOSC website to get various data about spins and masses. Subsequently, I made graphs of each parameter against merger time to see if any correlation exists. I found moderate, negative correlations between mass 1, mass 2, remnant mass, chirp mass, effective inspiral spin and average merger times, weak, negative correlations between mass ratios, percentage change in mass and average merger times, and a weak, positive correlation between final spin and average merger times. The weak correlations indicated that merger time was likely to be independent of change in mass, mass ratio and final spin.

## VIII. Acknowledgements

I would like to thank Professor Eric Myers for his guidance throughout the process of writing this paper. I also thank Pioneer Academics for this research opportunity. In addition, I would like to acknowledge the LIGO-Virgo collaboration for making their data public and accessible.

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