Significance of Optimal Matched Filtering in Detection of Gravitational Waves

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Introduction

Matter curves space-time. Non-spherical and/or non-uniform mass in motion, more precisely changing quadrupole mass distribution, is the source of ripples of curved space-time. These ripples that propagate away at the speed of light in space-time curvature are called gravitational waves [1]. The wave nature of light is used to measure the strength of these waves. Laser interferometer gravitational wave observatories use the interference ability of light, which is an electromagnetic wave. Warps in space-time caused by changing quadrupole mass distribution cause interference patterns to change, which allows its strength to be measured. However, due to the enormous distance they travel, it is very difficult to measure the signals of these weakened waves that are also extensively interfered with many different noise elements. Several computational methods were developed to eliminate those noises. One of them is optimal matched filtering, which makes a correlation between a signal template and the signal that is observed in the frequency domain (which is converted from the time domain with Fourier Transform). Matched filtering is optimal because it weakens the signal where noise is high and looking to where the signal-to-noise ratio is maximum.

The purpose of this study is to analyze the optimal matched filtering and to investigate how the filtering is affected when it is made non-optimal. In order to achieve this goal, the computer code was obtained from the Hardware Injection Tutorial [14] of the Gravitational Wave Open Science Center and slight changes were made. Data from the Gravitational Wave Open Science Center were analyzed using the code [2].

Gravitational waves (GW), one of the most energetic processes of the universe at the moment of their emergence, were first predicted more than a century ago by Albert Einstein, while he was formulating his general theory of relativity. The ripples that travel at the speed of light throughout the universe also carry valuable information about their origins and the nature of gravity [3].

Einstein has shown that accelerating neutron stars and black holes that are orbiting each other are disrupting space-time. Colliding black holes, collapsing supernovae, coalescing neutron stars and white dwarf stars are among the catastrophic events that produce gravitational waves [3].

The first proof of the existence of GW came in 1974 from two astronomers at UMass Amherst who discovered two extremely dense and heavy stars in orbit around each other. According to general relativity, this binary pulsar should radiate gravitational waves, a scenario very suitable to test Einstein's prediction. After eight years of observing changes in the period of the stars' orbit over time, astronomers determined that the stars were getting closer to each other at precisely the rate predicted by general relativity if they were emitting gravitational waves. Since then, this system has been monitored and the observed changes in the orbit precisely agree with general relativity. Hence, this proves that it is emitting gravitational waves [3].

 

Figure 1. The LIGO sites in Livingston, Louisiana (left) and Hanford, Washington (right) [4]

The first direct detection of gravitational waves (GW) that opened the possibility of observing our universe in gravitational waves was only made possible by the Laser Interferometer Gravitational-wave Observatory (LIGO) detectors located in the USA and operated by Caltech and MIT. LIGO (Fig. 1), one of the most sophisticated and sensitive scientific instruments ever built using extreme engineering and technology, represents the state of the art in GW science. Comprised of two laser interferometers thousands of kilometers apart from each other, LIGO uses the interference nature of the light like Michelson Interferometer (Fig. 2) in which a beam splitter splits laser beams to two different directions and the optical path differences between two arms cause the interference pattern to change [5].

At the end of the 19th century, scientists were thinking that like all other waves (sound, water, etc.), light should travel in a medium, too. They called that space matter “ether.” With the aim of proving the existence of ether, different observations in different directions made by Michelson and Morley have shown that light has a constant speed through empty space; those observations also determined that there is no kind of matter called “ether.” This failure is one of the biggest breakthroughs in the history of science. On the other hand, the same system has presented data on the existence of gravitational waves. Through gravitational waves, warps in space time occur that results in a change of length in one arm of the interferometer. This causes a change in the optical path of light in that arm and results in a change in the interference pattern (constructive or destructive), which leads to a change in the intensity of light that is perceived by sensors. Observations in LIGO measure the change in the interference pattern that occurs because of gravitational waves whose strength equals to the ratio of the changed length of the interferometer's arm to the total length of it, which is called strain amplitude ($h = \frac{\Delta L}{L}$) [6].

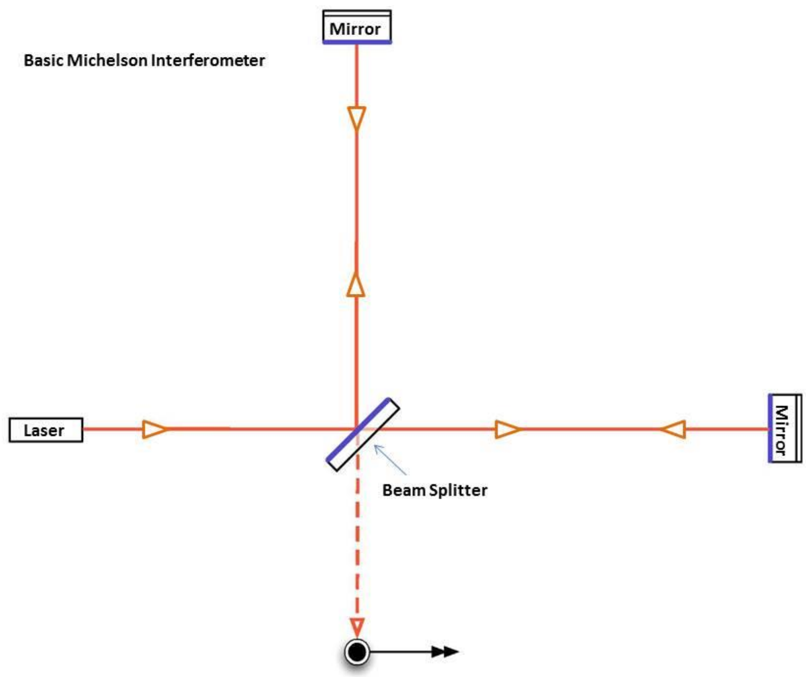

Figure 2. A beam splitter splits laser beams to two different directions, optical path differences between two arms cause interference pattern to change [6]

When this strain reaches LIGO its amplitude is on the degree of 10^{-20}. This extremely low-level signal is highly interrupted with diverse levels of noise (Fig. 3) including macroseismic and Newtonian to micro quantum level noises [7].

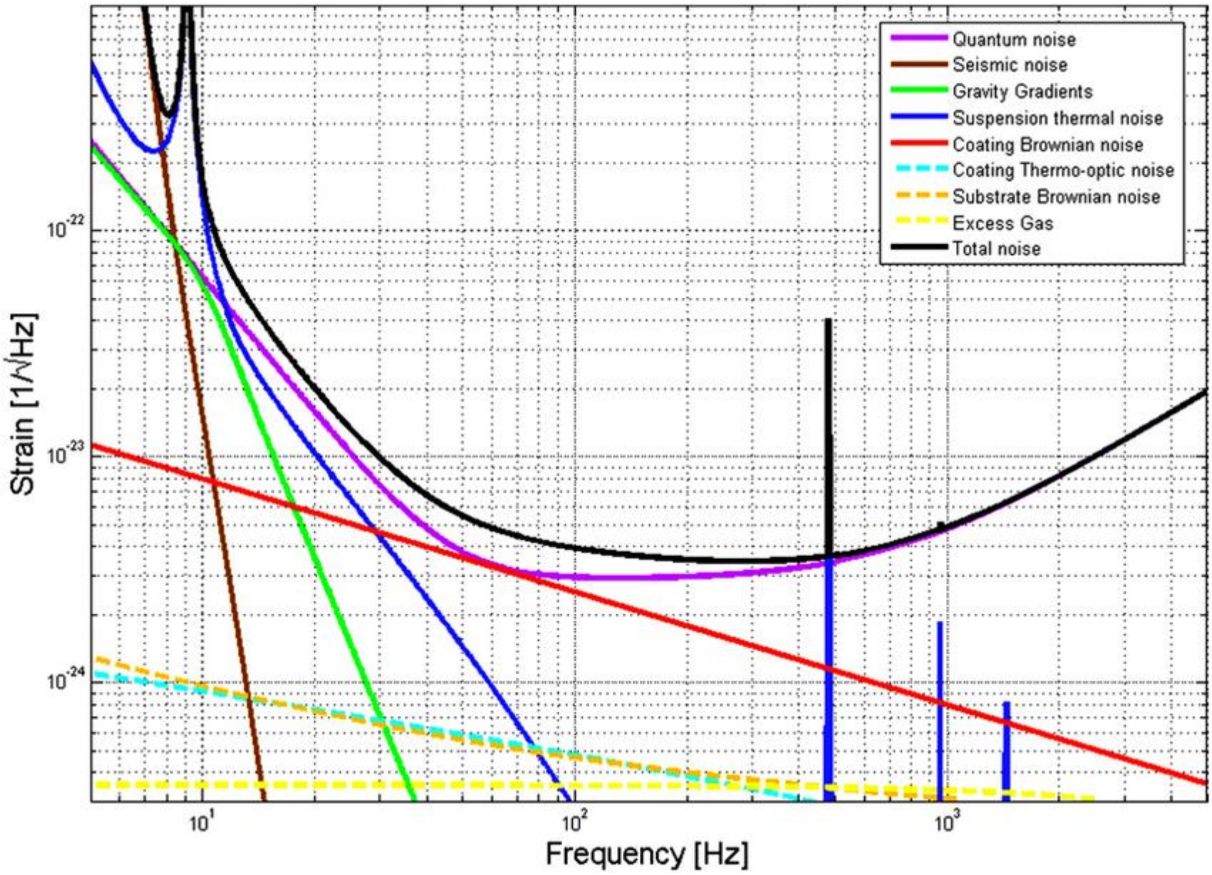

Figure 3. Sources of noises on amplitude spectral density [8]

In order to differentiate the true signal from these noises, three main methods are widely used: bandpass filtering, time domain cross-correlation and optimal matched filtering.

However, although the noise amplitude is much louder than that of the signal, it is still possible to detect the signal due to its higher power than the noise amplitude at some frequencies. In order to determine such frequencies, an Amplitude Spectral Density (ASD) of the template (for the signal) and the noise should be plotted. On the plot, the frequencies of the template that has the most power relative to the noise can be determined. The bandpass filtering method is used to cut off other frequencies and allows one to find where the signal is [9].

The second method is time domain cross-correlation, which multiplies the one second template by the whole signal starting from all seconds to see where the multiplication is. It can also be used after bandpassed data in order to increase precession [10].

The third method is optimal matched filtering. We know that, at some frequencies, there is more noise than others. To use this to our advantage, we are passing to a frequency domain using Fourier transform and making a frequency domain correlation while weighting each frequency by the inverse power of the noise amplitude and obtaining a signal-to-noise ratio (SNR). This basically increases the weight of a signal when noise is low and decreases the weight of a signal when noise is high [11].

Methods

Data was downloaded from the S5 Data Archive [12]. From the S5 Compact Binary Coalescence Hardware Injections [13] page, tables of injections were used to determine which injections will be used.

The Hardware Injection Tutorial [14] at the Gravitational Wave Open Science Center was used as the source of the code in this study. However, slight changes were also made. While noise is weighted with Power Spectral Density (PSD) during optimal matched filtering, all the values of all moments of PSD were summed and divided by the number of moments with noise and an average PSD was obtained. Thus, all the PSD values became the same and no weighting was done in the code. In total, this process was applied to five injections.

A similar procedure was also applied for the Binary Black Hole Event Tutorial [15], where the PSD of noise was changed by an average PSD, and four out of seven released events were tested.

Both injections and events were tested with optimal and non-optimal matched filtering methods.

Results

The results of all five injections tested were similar. Thus, two selected injections were further analyzed. Similarly, all events tested led to alike results. Thus, out of the four events tested, one was chosen for detailed analyses.

The first injection is obtained from H1, at 817061888 GPS Time, with a 4096 second long HDF5 file where the peak was observed on the 3011th second with 33.8 SNR when the optimal matched filtering was applied (Fig. 4, right). In contrast with the single dominant distinct peak of optimal matched filtering, when optimal matched filtering was not applied, many similarly sized peaks were observed. The highest SNR rate was 0.54 at roughly 3002nd seconds (Fig. 4, left).

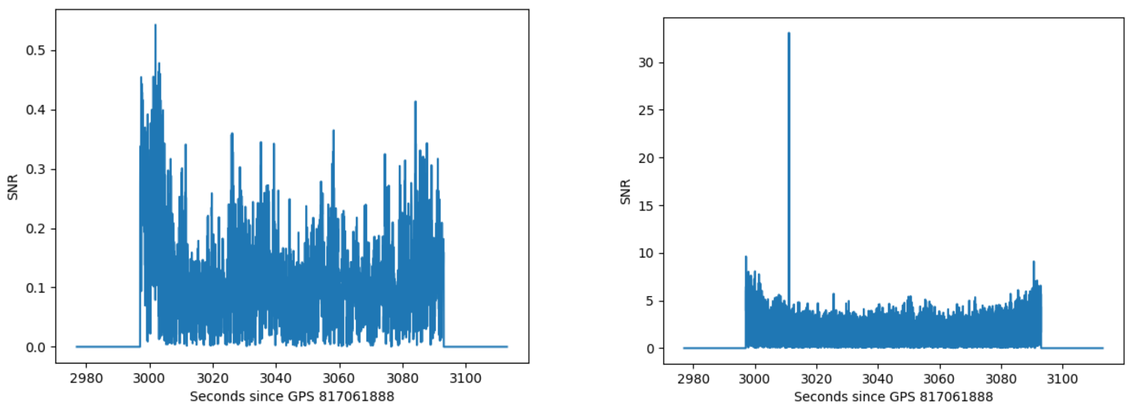

Figure 4. Non-optimal (left) and optimal (right) mached filtering for the 1st injection

The second injection is obtained from L1, at 823586816 GPS Time, with a 4096 second long HDF5 file in which the peak was observed on roughly the 3991th second with roughly 150 SNR when the optimal matched filtering was applied (Figure 5/right side). While optimal matched filtering generated a single clear peak, the non-optimal matched filtering has given many similarly sized peaks that lead to a strong background noise. The highest SNR rate obtained was 0.66 at roughly 4046th seconds (Fig. 5, left).

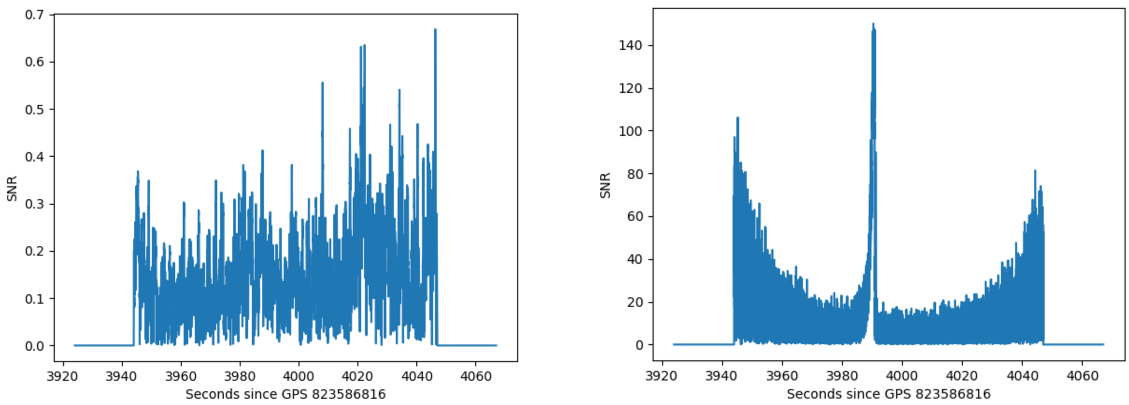

Figure 5. Non-optimal (left) and optimal (right) mached filtering for the 2nd injection

Related to the event tested (GW150914), in the situation where optimal matched filtering was applied, both of the detectors (L1 and H1) detected distinguishable peaks at second zero with roughly 13 SNR at L1 and 18.5 SNR at H1. Sharp peaks were apparent at both detectors with optimal matched filtering as they are clearly observed from the zoomed SNRs around the peaks (Fig. 6, bottom and Fig. 7, bottom). In the case when optimal matched filtering is not applied, distinct peaks were also missing at both the L1 and H1 detector results (Fig. 6, top and Fig, 7, top). Accordingly, L1 has given 11 SNR and H1 has given 42.5 SNR, both of which were on second zero. The peaks of both L1 and H1 were determined to be flatter as compared with the sharper peaks of the optimal matched filtering.

Figure 6. Non-optimal (top) and optimal (bottom) mached filtering for L1 for the event

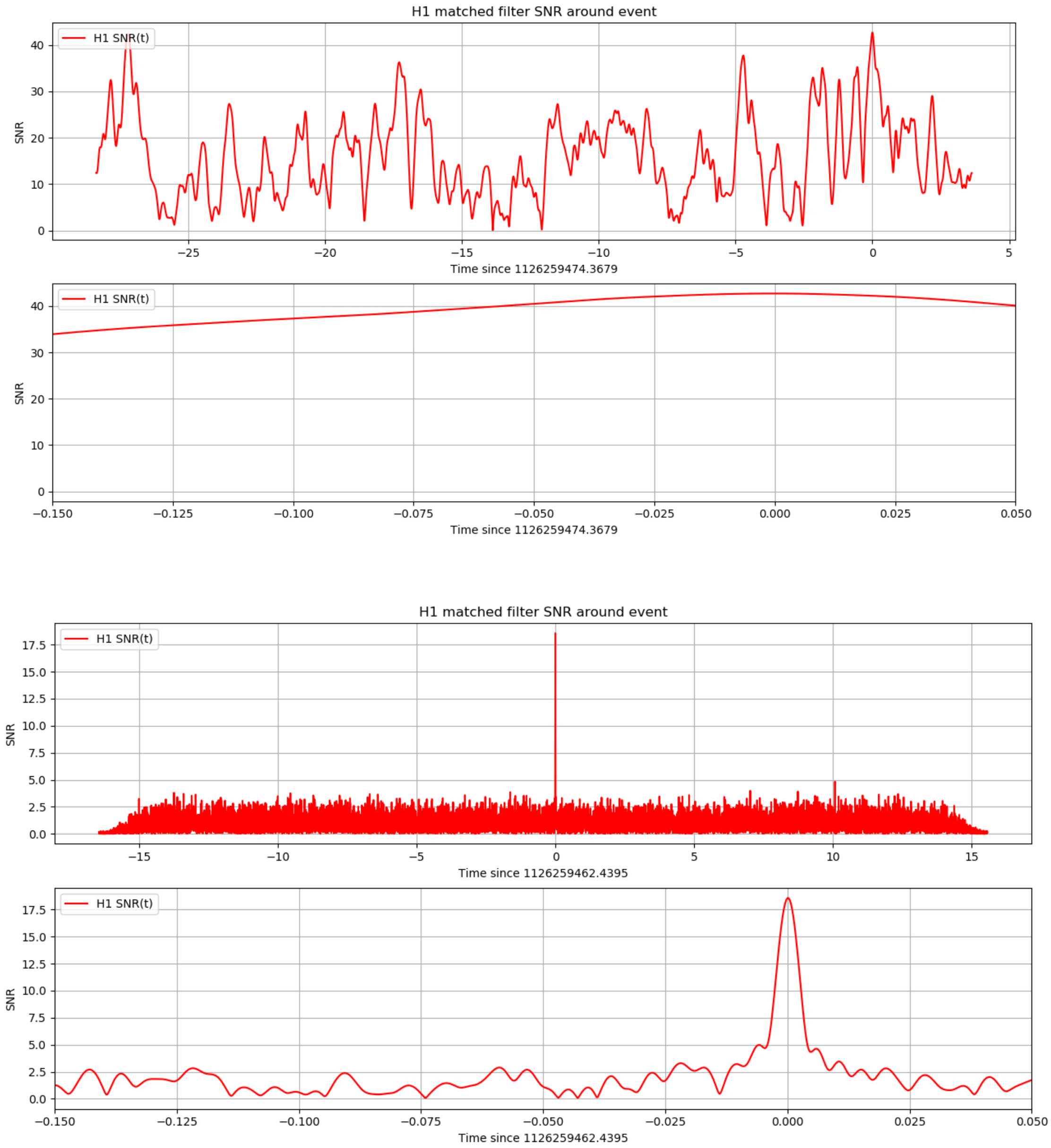

Figure 7. Non-optimal (top) and optimal (bottom) mached filtering for the H1 for the event

Conclusion

Optimal and non-optimal matched filtering was applied to five different injections and four distinct events. In all the injections, the highest SNR values were decreased to < 1 at higher levels and numerous indistinguishable peaks were observed. At every injection, non-optimal filtering gave a different and incorrect value as compared to the optimum matched filtering.

Though the number of peaks were also increased at the events and the result became uncertain, the highest peak always gave the same result with the optimal matched filtering. While the peak edges were sharp at optimum matched filtering, they were flattened at non-optimal matched filtering. Additionally, L1 and H1 were affected differently when non-optimal matched filtering is applied.

In conclusion, optimal matched filtering was found superior to non-optimal matched filtering in terms of generating a single peak that is clearly distinguished from the background noise. Further research may focus on questions related to the flattening of the peaks' edges and to facilitate our understanding by answering the question of why L1 and H1 are affected differently. Future studies may also enlighten us as to why injection always leads to deterioration, while a correct time (even if not precisely the same) is observed at the events.

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References

[1] Hartle, James B. *Gravity: an introduction to Einstein's general relativity*. San Francisco: Addison-Wesley. 2003.

[2] Gravitational Wave Open Science Center. https://www.gw-openscience.org (accessed August 2, 2019).

[3] “What are Gravitational Waves?” LIGO Caltech. https://www.ligo.caltech.edu/page/what-are-gw (accessed August 2, 2019).

[4] Mueller, Chris L. “Advanced LIGO.” Researchgate. https://www.researchgate.net/figure/The-LIGO-sites-in-Livingston-Louisiana-left-and-Hanford-Washington-right-The-arms_fig1_274265344 (accessed August 2, 2019).

[5] “What is LIGO?” LIGO Caltech. https://www.ligo.caltech.edu/page/what-is-ligo (accessed August 2, 2019).

[6] “What is an Interferometer?” LIGO Caltech. https://www.ligo.caltech.edu/page/what-is-interferometer (accessed August 2, 2019).

[7] “The Sensitivity of the Advanced LIGO Detectors at the Beginning of Gravitational Wave Astronomy.” LIGO Scientific Collaboration. https://www.ligo.org/science/Publication-O1Noise/index.php (accessed August 2, 2019).

[8] Luck, Harald. “AIGO: A Southern Hemisphere Detector for the Worldwide Array of Ground-based Interferometric Gravitational Wave Detectors.” Researchgate. https://www.researchgate.net/figure/Advanced-LIGO-modelled-noise-budget-Due-to-the-similarity-in-design-it-is-expected-that_fig3_40901511 (accessed August 2, 2019).

[9] “Find an Inspiral: Bandpass Filter.” Gravitational Wave Open Science Center. https://www.gw-openscience.org/tutorial_bandpass/ (accessed August 2, 2019).

[10] “Find an Inspiral: Time domain cross-correlation.” Gravitational Wave Open Science Center. https://www.gw-openscience.org/tutorial_crosscorr/ (accessed August 2, 2019).

[11] “Find an Inspiral: Optimal matched filter.” Gravitational Wave Open Science Center. https://www.gw-openscience.org/tutorial_optimal/ (accessed August 2, 2019).

[12] “Archive for S5 Dataset.” Gravitational Wave Open Science Center. https://www.gw-openscience.org/archive/S5/ (accessed August 2, 2019).

[13] “S5 Compact Binary Coalescence Hardware Injections.” Gravitational Wave Open Science Center. https://www.gw-openscience.org/s5hwcbc/ (accessed August 2, 2019).

[14] “Find a Hardware Injection: Step 1.” Gravitational Wave Open Science Center. https://www.gw-openscience.org/inj/step1/ (accessed August 2, 2019).

[15] “Binary Black Hole Signals in LIGO Open Data.” Gravitational Wave Open Science Center. https://www.gw-openscience.org/GW150914data/ LOSC_Event_tutorial_GW150914.html (accessed August 2, 2019).