Logarithms, Exponents, and Logarithmic Scales

How to wrap your mind around incredibly large or small numbers

Multiplication

$10 = 10^1$ so $\log_{10}(10) = 1$

$100 = 10^2$ so $\log_{10}(100) = 2$

$1000 = 10^3$ so $\log_{10}(1000) = 3$

$10,000,000 = 10^7$ so $\log_{10}(10,000,000) = 7$

$$10^3 \times 10^7 = 1,000 \times 10,000,000 = 10,000,000,000 = 10^{10}$$

$$\log_{10}(10^3) + \log_{10}(10^7) = \log_{10}(10^{10}) = 10$$

➔ When you multiply numbers, you add their logarithms.

Example

$1.496 = 10^{0.174931}$ so $\log_{10}(1.496) = 0.174931$

$\log_{10}(1.496 \times 10^{11}) = \log_{10}(1.496) + \log_{10}(10^{11})$

$= 11 + 0.174931$

$= 11.174931$

Negative logarithms

$0.1 = 10^{-1}$ so $\log_{10}(0.1) = -1$

$001 = 10^{-2}$ so $\log_{10}(0.01) = -2$

$0.001 = 10^{-3}$ so $\log_{10}(0.001) = -3$

$0.0000001 = 10^{-7}$ so $\log_{10}(0.0000001) = -7$

$10^{-3} \times 10^{7} = 0.001 \times 10{,}000{,}000 = 1\ 0{,}000 = 10^{4}$

$\log_{10}(10^{-3}) + \log_{10}(10^{+7}) = \log_{10}(10^{4}) = 4$

Exponent

$$100 \times 100 \times 100 = 100^3 = 10^6$$

so

$$\log_{10}(100^3) = 3 \times \log_{10}(100)$$
$$= 3 \times 2$$
$$= 6$$

➔ When you raise a number to a power,
you multiply the logarithm by the power.

$$\log_{10}(x^n) = n \log_{10}(x)$$

Logarithmic Scales

Each increase in the power of 10 is called a "cycle".

When you plot data on a log-log graph, a straight line indicates a power law.

The slope is the power law exponent

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$